Accommodate chiral symmetry breaking and linear confinement in a dynamical holographic QCD model

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Abstract. We construct a self-consistent holographic QCD model which can realize two most important phenomena of QCD, i.e. chiral symmetry breaking and confinement. The model is formulated in the framework of graviton-dilaton-scalar system, where the dilaton field is of dimension-2 which might be dual to the dimension-2 gluon condensate and can lead to the linear confinement, while the scalar field corresponds to the quark anti-quark condensate and can explain the property of chiral dynamics. Within this framework, both Regge spectra of hadrons and the linear potential between quarks can be accommodated. It is also found that the negative dilaton background can be safely excluded in this framework.

INTRODUCTION

It is well-known that the QCD vacuum is characterized by spontaneous chiral symmetry breaking and color charge confinement. The spontaneous chiral symmetry breaking is well understood by the dimension-3 quark condensate $\langle \bar{q}q \rangle$ [1] in the vacuum, in spite of that, the understanding to confinement remains a challenge. Confinement can be reflected by the Regge trajectories of hadrons [2], which suggests that the color charge can form the string-like structure inside hadrons. It can also be shown by the linear potential between two quarks (either light or heavy) at large distances, *i.e.* $V_{\bar{Q}Q}(R) = \sigma_s R$ with σ_s the string tension.

The discovery of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence and the conjecture of the gravity/gauge duality [3] provides a revolutionary method to tackle the problem of strongly coupled gauge theories. Many efforts have been invested in searching for such a realistic description by using the "bottom-up" approach, e.g. see Ref. [4] for reviews. The Regge trajectories and linear quark potential are like two sides of a coin, both describe the properties of linear confinement. A successful holographic QCD model should describe both the Regge trajectories of hadron spectra and linear quark potential. Nonetheless, the models on market cannot accommodate both. Currently a working framework used to describe the Regge trajectories of hadron spectra is the soft-wall AdS_5 model or Karch-Katz-Son-Stephanov (KKSS) model [5] and its extended version [6, 7, 8], where a quadratic dilaton background is introduced

in the 5D action. However, with AdS_5 metric, only Coulomb potential between the two quarks can be produced [9]. The working holographic QCD model to realize the linear quark potential was proposed in Ref.[10], where a positive quadratic correction in the deformed warp factor of AdS_5 geometry was introduced. (The linear heavy quark potential can also be obtained by introducing other deformed warp factors as shown in Refs. [11, 12].) The positive quadratic correction in the deformed warp factor in some sense behaves as a negative dilaton background in the 5D action, which motivates the proposal of the negative dilaton soft-wall model [13, 14]. More discussions on the sign of the dilaton correction can be found in [15, 16].

It is interesting to explore how to generate both the linear Regge behavior of hadron spectra and linear quark potential in a self-consistent model. In this work, a dynamical holographic QCD model is proposed and formulated in the graviton-dilaton-scalar coupled system [17].

GRAVITON-DILATON-SCALAR COUPLED SYSTEM

We first briefly describe the graviton-dilaton coupled system, where the dilaton background is expected to be dual to the effective degree of freedom in the pure gluon system. In general, a dilaton background will deform the warp factor of the metric structure [18, 19, 20]. The 5D action of the graviton-dilaton system is defined as

$$S_{GD} = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\phi} \left(R + 4\partial_m \phi \partial^m \phi - V_\phi \right). \tag{1}$$

Where G_5 is the 5D Newton constant, g_s, ϕ and V_ϕ are the 5D metric, the dilaton field and dilaton potential in the string frame, respectively. Under the quadratic dilaton background $\phi = \mu^2 z^2$, the analytic solution of the dilaton potential in the Einstein frame $V_\Phi^E = e^{4\phi/3} V_\phi$ with $\phi = \sqrt{\frac{3}{8}}\Phi$ takes the form of

$$V_\Phi^E = -12 \frac{{}_0F_1(1/4; \frac{\Phi^2}{24})^2}{L^2} + 2 \frac{{}_0F_1(5/4; \frac{\Phi^2}{24})^2 \Phi^2}{L^2}, \tag{2}$$

here L the radius of AdS_5 and ${}_0F_1(a;z)$ the hypergeometric function. In the ultraviolet limit, $V_\Phi^E \xrightarrow{\Phi \to 0} -\frac{12}{L^2} + \frac{1}{2}M_\Phi^2 \Phi^2$ with the 5D mass for the dilaton field $M_\Phi^2 L^2 = -4$. From the AdS/CFT dictionary $\Delta(\Delta - 4) = M_\Phi^2 L^2$, one can derive its dimension $\Delta = 2$. The most likely dimension-2 operator candidate in QCD is A_μ^2. It has been pointed out in Ref.[21] that the dimension-2 gluon condensate plays essential role for the linear confinement. This A_μ^2 can be put into a gauge invariant form [22], which might be related to topological defects in QCD vacuum [23]. The quadratic dilaton field might be dual to the dimension-2 gluon condensate, and the graviton-dilaton system describes the pure gluodynamics.

We now add the probe of flavor dynamics on the pure gluodynamic background, and extend the graviton-dilaton system to the framework of graviton-dilaton-scalar coupling system, where the scalar field captures chiral dynamics. The graviton-dilaton-scalar

system can be described by the following 5D action:

$$S = S_{GD} + S_M, \tag{3}$$

with S_{GD} given in Eq.(1) and S_M the KKSS action for mesons as in [5] taking the form of

$$S_M = -\frac{N_f}{N_c} \int d^5x \sqrt{g_s} e^{-\phi} Tr\left(|DX|^2 + V_X + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right). \tag{4}$$

Where X and V_X are the scalar field and its corresponding potential. g_5^2 is fixed as $12\pi^2 N_f/N_c^2$ [5] and we take $N_f = 2, N_c = 3$ in this paper.

We assume the vacuum background is induced by the dilaton field of dimension-2 gluon condensate and the scalar field of the quark antiquark condensate $<X>= \frac{\chi(z)}{2}$ [5], then the vacuum background part of the action Eq.(3) takes the following form

$$
\begin{aligned}
S_{vac} &= \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} \{ e^{-2\phi}\left(R_s + 4\partial_m\phi\partial^m\phi - V_\phi\right) \\
&- \lambda e^{-\phi}\left(\frac{1}{2}\partial_m\chi\partial^m\chi + V_\chi\right) \},
\end{aligned} \tag{5}
$$

with $\lambda = \frac{16\pi G_5 N_f}{N_c L^3}$ and the metric in the string frame

$$dS_s^2 = B_s^2(-dt^2 + d\vec{x}^2 + dz^2), \quad B_s^2 \equiv e^{2A_s} \equiv L^2 b_s^2. \tag{6}$$

It is easy to derive the following three coupled field equations:

$$-A_s'' + A_s'^2 + \frac{2}{3}\phi'' - \frac{4}{3}A_s'\phi' - \frac{\lambda}{6}e^\phi\chi'^2 = 0, \tag{7}$$

$$
\begin{aligned}
&\phi'' + (3A_s' - 2\phi')\phi' - \frac{3\lambda}{16}e^\phi\chi'^2 \\
&- \frac{3}{8}e^{2A_s - \frac{4}{3}\phi}\partial_\phi\left(e^{4/3\phi}V_\phi + \lambda e^{7/3\phi}V_\chi\right) = 0, \tag{8}
\end{aligned}
$$

$$\chi'' + (3A_s' - \phi')\chi' - e^{2A_s}\partial_\chi V_\chi = 0. \tag{9}$$

If we know the dynamical information of the dilaton field ϕ and the scalar field χ, then the metric A_s, the dilaton potential V_ϕ and the scalar potential V_χ should be self-consistently solved from the three coupled equations given in Eqs.(7,8,9). It is noticed that the graviton-dilaton-scalar system is different from the graviton-dilaton-tachyon system [24], where the metric remains as AdS$_5$.

Constraints from chiral symmetry breaking and linear confinement

As proposed in the KKSS model, at ultraviolet(UV), the scalar field takes the following asymptotic form,

$$\chi(z) \xrightarrow{z\to 0} m_q\zeta z + \frac{\sigma}{\zeta}z^3, \tag{10}$$

where m_q is the current quark mass, and σ is the quark antiquark condensate, and ζ is a normalization constant and is fixed as $\zeta^2 = \frac{N_c^2}{4\pi^2 N_f}$. In this paper, we would fix $m_q = 5\text{MeV}, \sigma = (228\text{MeV})^3$.

The linear behavior of quark-antiquark static potential in the heavy quark mass limit $m_Q \to \infty$ can describe the permanent confinement property of QCD. Following Ref. [9], one can solve the renormalized free energy of the $\bar{q}q$ system under the general metric background A_s. One can find that at the point z_c when $b_s'(z_c) \to 0$,

$$\frac{V_{\bar{q}q}(z_0)}{R_{\bar{q}q}(z_0)} \xrightarrow{z_0 \to z_c} \frac{g_p}{2\pi} b_s^2(z_c). \tag{11}$$

Here $g_p = \frac{L^2}{\alpha}$ and α the 5D effective string tension. Therefore, the necessary condition for the linear part of the $q - \bar{q}$ potential is that there exists one point z_c or one region, where $b_s'(z) \to 0, z \to z_c$ while $b_s(z)$ keeps finite. For simplicity, we can take the following constraint on the metric structure at IR: $A_s'(z) \xrightarrow{z \to \infty} 0, A_s(z) \xrightarrow{z \to \infty}$ Const. Under these conditions, the equation of motion Eq.(7) in the IR takes a simple form: $\frac{2}{3}\phi'' - \frac{\lambda}{6}e^\phi \chi'^2 = 0$, which provides a relation between the chiral condensate and dimension-2 gluon condensate at IR. The asymptotic form of $\chi(z)$ at IR can be solved as:

$$\chi(z) \xrightarrow{z \to \infty} \sqrt{8/\lambda}\mu e^{-\phi/2}. \tag{12}$$

To match the asymptotic forms both at UV and IR in Eqs.(10) and (12), χ can be parameterized as

$$\chi'(z) = \sqrt{8/\lambda}\mu e^{-\phi/2}(1 + c_1 e^{-\phi} + c_2 e^{-2\phi}) \tag{13}$$

with $c_1 = -2 + \frac{5\sqrt{2\lambda}m_q\zeta}{8\mu} + \frac{3\sqrt{2\lambda}\sigma}{4\zeta\mu^3}$ and $c_2 = 1 - \frac{3\sqrt{2\lambda}m_q\zeta}{8\mu} - \frac{3\sqrt{2\lambda}\sigma}{4\zeta\mu^3}$.

Regge trajectories of mesons

Under the positive quadratic dilaton background and the scalar profile Eq.(13), the metric structure $A_s(z)$ or $b_s(z)$ can be solved through Eq.(7). Considering the meson fluctuations under the vacuum background described by A_s, ϕ, χ as in [5], we can split the fields into background part and perturbation part. For X we would have a scalar perturbation s and pseudo-scalar perturbation π, i.e., $X = (\frac{\chi}{2} + s)e^{i2\pi^a t^a}$. For the vector field V_μ and axial vector field A_μ part, due to their vanishing vacuum expectation value, we would use the same notation to denote the perturbation fields. The equations of motion for perturbation fields s, π, V_μ, A_μ can be easily derived. For example, the schrodinger like equation for vector is given below:

$$-v_n'' + V_v(z)v_n = m_{n,v}^2 v_n, \tag{14}$$

$$V_v(z) = \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4}. \tag{15}$$
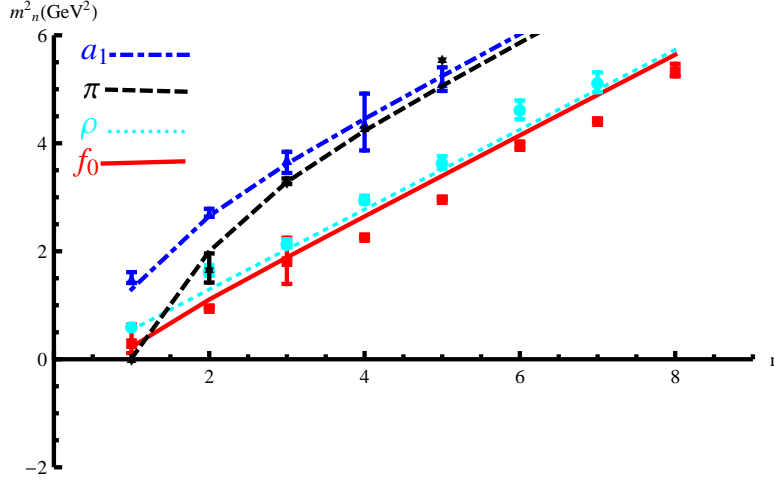
FIGURE 1. A plot of experimental(dot) and model predicted (line) mass square spectra for the scalar and pseudoscalar mesons f_0, π and vector and axial-vector mesons ρ, a_1.

It is noticed that at IR, $V_v(z) = -\mu^2 + \mu^4 z^2$, therefore the solution of Eq.(15) for high excitations is $M_n^2 = 4\mu^2 n$.

By fixing the 5D Newton constant $G_5 = \frac{3L^3}{4}$, one can produce the proper splitting between the vector and axial vector Regge trajectories. The produced spectra of scalar f_0, pseudoscalar π, vector ρ and axialvector a_1 are shown in Fig.1 compared with experimental data [25]. The experimental data for f_0 are chosen as in Ref.[6]. With only 4 parameters, all produced meson spectra in the graviton-dilaton-scalar system agree well with experimental data. We would emphasize that our model can incorporate both chiral symmetry breaking and confinement properties in the hadron spectra, and the slope of Regge trajectories is $4\mu^2$ with $\mu = 0.43$GeV. In a similar dynamical holographic QCD model in Ref. [26], the linear Regge behavior is realized but the chiral symmetry breaking mechanism is missing.

The string tension of the linear quark potential.

Under the metric background $A_s(z)$ solved from Eq.(7) with the positive quadratic dilaton background and the scalar profile Eq.(13), the quark potential can be solved numerically. In the UV limit, one can derive the Coulomb potential $V_{\bar{q}q} = -\frac{0.23 g_p}{R_{\bar{q}q}}$ as given in Ref.[9]. In the IR limit, we can get the linear potential $V_{\bar{q}q} = \frac{g_p}{2\pi} b_s^2(z_c) R_{\bar{q}q}$. From the solutions in Eq.(7), we have $b_s^2 \approx 4\mu^2$, which indicates that the string tension of the linear quark potential $\sigma_s \sim 4\mu^2$. The numerical result for the quark potential $V_{\bar{q}q}$ as a function of quark anti-quark distance R_{qq} is shown by the solid line in Fig.2. The two parameters are fixed as $g_p = 2.3$ and $\mu = 0.43$GeV. The result agrees with the Cornell potential (dot-dashed line) [27] $V^c(R) = -\frac{\kappa}{R} + \sigma_{str} R + V_0$ with $\kappa \approx 0.48$, $\sigma_{str} \approx 0.183$GeV^2 and $V_0 = -0.25$GeV.
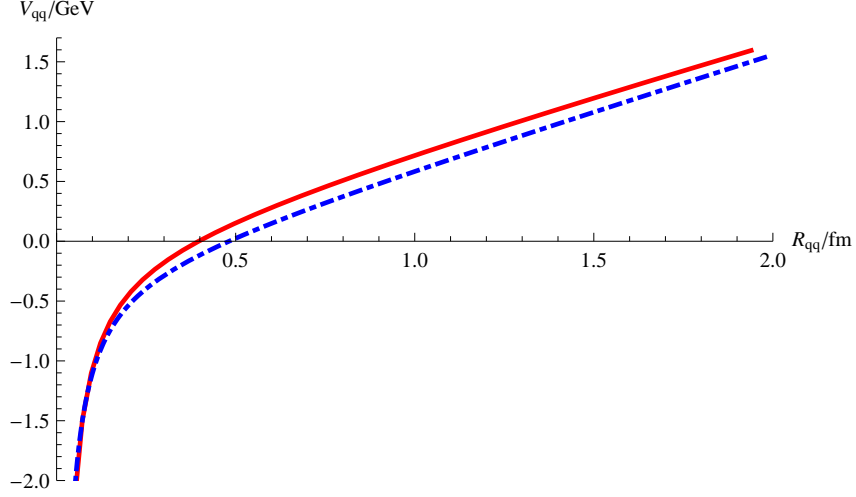
FIGURE 2. $V_{\bar{q}q}$ as a function of $R_{\bar{q}q}$ from our model (solid line) with $g_p = 2.3$ and $\mu = 0.43\text{GeV}$ compared with the Cornell potential (dot-dashed line).

The sign of the dilaton field

The last but not the least, we address the issue of the sign of the dilaton background. If we choose a negative dilaton background $\phi = -\mu^2 z^2$ as in Ref.[13, 14], in the IR limit, the last term in Eq.(7) decreases exponentially to zero, and one can get the asymptotic solution of $A_s(z) \xrightarrow{z\to\infty} -\frac{4}{3}\mu^2 z^2 + 1/2\log(z)$ and $b_s(z) \sim \sqrt{z}e^{-\frac{4}{3}\mu^2 z^2} \xrightarrow{z\to\infty} 0$. From Eq.(11), it's not possible to produce the linear potential with a negative dilaton background. Therefore, one can safely exclude the negative dilaton background in the graviton-dilaton-scalar system.

CONCLUSION

In summary, we propose a dynamical holographic QCD model, which takes into account the back-reaction of flavor dynamics on the pure gluodynamic background. To our knowledge, this is the first dynamical holographic QCD model which can produce both the linear Regge trajectories of hadron spectra and quark anti-quark linear potential. It is observed that both the slope of the Regge trajectories and the string tension of the linear quark anti-quark potential are proportional to the dimension-2 gluon condensate. This result indicates that the linear confinement is dynamically induced by the dimension-2 gluon condensate. The holographic QCD model offers us a new viewpoint on the relation between the chiral symmetry breaking and confinement. It is found that the balance between the chiral condensate and dimension-2 gluon condensate is essential to produce the correct Regge behavior of hadron spectra. As a byproduct, it is found that the negative dilaton background can be safely excluded in the framework of graviton-dilaton-scalar system.

ACKNOWLEDGEMENTS

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